5.1 *Quasi-momentum operator of Bloch electrons*.

As mentioned in the class, the quantum-mechanical momentum operator $\hat{p} = -i\hbar\nabla$ does not commute with the effective single-electron Hamiltonian of a solid, $\widehat{H} = -\frac{\hbar^2\nabla^2}{2m} + U(\vec{r})$, where $U(\vec{r}+\vec{R}) = U(\vec{r})$ with $\vec{R}$ a lattice vector. However, it is possible to construct a quasi-momentum operator $\widehat{P}$ that commutes with $\widehat{H}$ and whose eigenvalue is the quasi-momentum (or crystal momentum) $\hbar k$, so that

$$[\widehat{H}, \widehat{P}] = 0, \qquad \widehat{P}\psi_k(\vec{r}) = \hbar\vec{k}\psi_k(\vec{r}),$$

where $\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_k(\vec{r})$ is the Bloch function. Since $\widehat{P} \to \hat{p}$ for $U(\vec{r}) = const$, we can express quasi-momentum operator $\widehat{P}$ as $\widehat{P} = \hat{p} + i\hbar\widehat{F}$.

(1) Evaluate the commutator $[\widehat{H}, \hat{p}]$.

(2) Find the unknown operator $\widehat{F}$.

习题

5.2 阎书3.1：电子在周期场中的势能函数

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2[b^2-(x-na)^2], na-b \le x \le na+b \\ 0, (n-1)a+b \le x \le na-b \end{cases}$$

其中a=4b，ω为常数，
（1）画出此势能曲线，并求其平均值；
（2）用近自由电子近似模型求出晶体的第一个和第二个禁带的宽度。

5.3 黄书4.1

5.4 黄书4.2